Accessibility Strategies for Mathematics

"Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students."

-- Principles and Standards for School Mathematics (NCTM, 2000, p.12)

This document provides an organized list of strategies that teachers can use to make mathematics more accessible to students with disabilities. The goal is to enable teachers to provide support so students with learning disabilities can succeed, while maintaining high standards and the integrity of the mathematics.

The Addressing Accessibility in Mathematics (AAM) group examined current research on student difficulties in mathematics and analyzed the kinds of tasks students are asked to use in various middle school mathematics curricula. Based on this, analysis AAM identified six areas in which students' strengths and needs strongly affect mathematics learning. The lists that follow detail the types of tasks commonly required in the six areas, along with examples of student difficulties and corresponding accessibility strategies. Note that some mathematics problems, such as multi-step word problems, involve tasks from multiple areas.

Many of the strategies provide *scaffolding* so that students can focus on the main mathematical content. For example, a strategy might remove a mechanical aspect of a task, such as drawing a table, so students can focus on higher-order thinking and demonstrate their grasp of concepts. Over time, the scaffolding is often removed—therefore, part of planning accessibility strategies is considering how and when to remove the scaffolding. It's akin to learning to ride a bicycle: at first, training wheels help a child focus on riding without having to worry about falling over.

Some students may always need certain supports; others may leave the supports behind. In either case, the students can expand their own repertoires of strategies, building on their strengths to help bypass their weaknesses.

The eight areas addressed in this document:

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The lists of strategies are not all inclusive and we welcome your suggestions! If you have a strategy that you would like to add, please email us at abrodesky@edc.org.

Conceptual

Standards-based mathematics emphasizes the need to build a deep understanding of concepts. This involves making connections among mathematical ideas, facts, and skills, and reflecting upon and refining one's own understanding. In middle school, students begin to work with abstract concepts such as variables and make greater use of symbolic representations. Students who tend to think concretely may need additional support in order to move from concrete to abstract representations.

Conceptual		
Task Demands	Examples of Student Difficulties	Accessibility Strategies to Consider
Use or manipulate symbols	 Does not always connect symbols with what they represent Does not remember the meaning of symbols 	 Use manipulatives to connect symbols to concrete objects Post wall charts or provide resource sheets with symbols and meanings
Solve abstract problems	 Does not understand abstract problems easily Tends to think concretely 	 Set up the investigation so that students move from the concrete to semi-concrete to the abstract Make connections to familiar contexts
Visualize and extend patterns	 Has difficulty visualizing and identifying patterns 	 Use manipulatives to build and extend patterns Provide simpler patterns
Make generalizations	 Finds it difficult to make generalizations and to write rules Tends to think concretely 	 Provide generalizations for students to test Have students use multiple representations of situation and then make a generalization
Understand mathematical relationships and make connections	• Thinks of mathematics as disparate parts and doesn't see the connections	 Make explicit connections between current and prior lessons or units Use concept maps
Learn, represent, and explain new concepts	 Tends to think concretely Focuses on small parts and does not see big picture Does not identify key points 	 Use hands-on investigations to build understanding Contrast examples and non-examples of a concept Provide resource sheets with summary information on complex concepts Use frequent assessments to identify gaps in the students' understanding of concepts Use multiple representations of concepts Make concept maps Provide organizers for students to complete Use concept map software like Inspiration
Apply concepts to new situations	 Sees new problems as unfamiliar Does not see a connection between new problems and those he or she has already solved 	 Help students to see the connections between new problems and prior work
Self-monitor understanding and ask clarifying questions	• Lacks a metacognitive awareness of what he/she doesn't understand	 Have students to reflect on their own learning using questions from KWL strategy: "What do I Know? What do I Want to learn? What have I Learned?"

Language

In mathematics, students need to describe strategies, explain their reasoning, justify solutions, and make persuasive arguments, both orally and in writing. They need to learn mathematical vocabulary and use it to express ideas with precision and clarity. In class and small group discussions, they need to build on the thinking of their classmates and to ask questions to help them understand another person's strategies.

Language		
Task Demands	Examples of Student Difficulties	Accessibility Strategies to Consider
Read directions and problems	 Has difficulty decoding words Reads slowly 	 Read aloud Use a tape recorder (or use taped texts from <i>Recordings for Blind and Dyslexic</i>) Digitize materials and use text-to-speech software
	 Finds comprehension challenging Tends to misunderstand directions 	 Have students highlight key points and identify unnecessary information Use pre-reading questions to focus their attention
Follow verbal directions	 Has difficulty with the auditory processing of verbal information Does not understand verbal directions well 	 Provide written as well as oral directions Make handouts of the overhead masters Have students rephrase directions in their own words Use an overhead
Write explanations of mathematical thinking	 Takes a long time to get started on writing tasks 	 Reword the question as a statement for students to complete Have students talk about ideas with a partner before writing them down
	 Does not know how to organize ideas 	 Use graphic organizers and writing templates, such as paragraph templates Use the same writing process as Language Arts Teach organizational strategies Use outlining software such as Inspiration
	• Gets distracted rather than focusing on the writing task	 Break writing tasks into smaller parts and provide frequent feedback
	 Does not have necessary fine-motor skills for extended writing 	 Have the student dictate to a "scribe" Use a computer or portable keyboard such as Alpha-smart Have the student record ideas on a tape recorder
Participate in Class Discussions	Does not express ideas orally with ease	 Prearrange when you will call on the student or use a nonverbal signal
	 Does not listen well to other students' explanations and gets distracted easily 	 Reduce the time for whole group discussions. Break class into small discussion groups and then have groups report back to the whole group
Give Oral Presentations	 Is not comfortable speaking in front of class Speaks slowly 	 Provide an organizer with questions for preparing the talk Provide practice time

Visual-Spatial

Representing mathematical ideas is key to understanding mathematics. Students use representations to solve problems, explore concepts, and communicate ideas. For example, students use different visual representations for percentages, including number lines, fraction circles and hundred-grids. In algebra, students use visual patterns to determine rules, analyze graphical representations of functions, and create mathematical models. Some difficulties with such tasks are caused by a breakdown in the processing of visual information. Students may benefit from such strategies as color-coding systems to help them focus on key information, and from learning explicit strategies for interpreting visual representations.

Visual-Spatial Processing		
Task Demands	Examples of Student Difficulties	Accessibility Strategies to Consider
Create and interpret visual representations	 Has difficulty representing mathematics concepts visually Does not connect graphics to the concepts they represent 	 Provide handouts of the representations for students to draw on, highlight, measure, and cut out Provide manipulatives
	 Finds it difficult to visualize and represent a three-dimensional model in two dimensions Finds it difficult to interpret a two- dimensional representation of a three-dimensional model 	 Provide examples of actual 3-D models for students to view or manipulate
Work with tables and graphs	 Has difficulty figuring out how to create tables or graphs or has difficulty physically creating them Has difficulty reading or interpreting graphs 	 Provide templates that address particular needs (for example, larger or partially filled-in tables) Use scaffolding strategies to help students eventually develop their own templates for tables, charts, and graphs
Read text	• Cannot read standard-size text	 Use larger fonts Provide oral versions (spoken, taped) of the instructions and text, where appropriate Use text-to-speech software Provide Braille version of the text
Read handouts and book pages	• Finds crowded pages distracting	 Reorganize the material into a handout Make all of the handouts single-sided and provide ample white space
	 Has difficulty focusing on the important information Finds extraneous material distracting 	 Have students highlight the key information Eliminate extraneous page features Explicitly teach how to find information in a book, noting chapter structures, bold text, previews, and summary boxes In preparing materials, consistently use methods such as bolding or underlining

Copy or read information displayed on a blackboard, chart, or overhead	 Does not see board well Does not know where to focus 	 Use large font sizes for overhead masters and give copies of the masters as handouts Seat students close to the blackboard Reduce glare from the windows Use a consistent format for displaying information on the board Use color coding
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Organization

Problem solving is integral to mathematical learning. The NCTM Problem Solving standard states that "students should have frequent opportunities to formulate, grapple with and solve complex problems that require a significant amount of effort." (NCTM, 2000) Complex problems make many organizational demands—students must figure out how to get started; carry out a sequence of steps; keep track of the information from prior steps; monitor their own progress and adjust strategies accordingly; and present solutions in an organized manner. Further, they must organize their time to ensure that they neither rush through tasks and make careless errors, nor spend too much time and yet not complete the task.

Organization		
Task Demands	Examples of Student Difficulties	Accessibility Strategies to Consider
Solve multi-step or complex problems	 Has a hard time getting started 	Provide hints or promptsTeach problem-solving strategies
	 Does not know how to figure out a sequence of steps for solving the problem 	 Teach organizational strategies such as breaking the problem into smaller parts Give frequent feedback
	 Rushes through tasks or spends excessive time 	 Teach organizational strategies for using time wisely Teach students to pause at specific points to check work Remind students of how much time remains for completing the tasks
	 Does not answer all of the questions or all parts of the investigation 	 Explicitly teach about the layout of the book and the question formats Provide a handout of the questions that students can highlight or underline
Make a table, graph, chart, number-line, spinner, or map	• Gets confused by the multiple steps involved in making a table, graph, etc.	• Provide Resource Sheets that list the steps involved or provide examples or templates
Collect and record data	 Records data in a disorganized manner that is difficult to analyze Has difficulty organizing data into tables 	Use table templates for data collection
Find information in prior student work	• Does not organize class notes well and thus has trouble finding the needed information	• Use a notebook organization system and reinforce it with notebook checks (if possible, use the same notebook organization system across subject areas)
Complete long-term assignments or projects	 Has difficulty organizing a large project Works slowly or spends an excessive amount of time Does not manage project resources well Needs help breaking a large task into steps 	• Provide a Project Organizer in which the project is broken into steps with due dates. Establish frequent check-in points.

Memory

Memory plays an important role in learning mathematics. For example, students use their memories to perform calculations and procedures, identify geometric figures, and create graphs that have all of the necessary parts.

- **Short-term memory** can only hold small amounts of information for a brief amount of time (seconds). After information enters short –term memory, you need to decide what to do with it. Short-term memory difficulties can affect a student's ability to copy info from the board, to take notes, follow multi-step directions, follow a presentation, or add to a class discussion.
- **Working memory** involves holding information in mind while you use it, such as performing mental calculations for a multi-step problem. Working memory holds information for seconds to hours and then may move some information into long-term memory. Students may lose track of steps when solving a multi-step problem and have difficulty remembering what they are reading.
- Long-term memory involves storing information for long-term use so that it can be retrieved later. Students with long-term memory deficits may not easily store information in memory, or may have difficulty retrieving information. Long-term memory difficulties can affect students' abilities to remember math facts with accuracy and fluency, to use mathematical vocabulary and to make connections among concepts that they have learned in prior months or years.

Memory	Student Difficulties	Associative Strategies to Consider
Task Demands Use math facts with accuracy and fluency	 Student Difficulties Has difficulty memorizing or recalling basic facts Retrieves incorrect facts Lacks fluency: recall of facts is slow and inaccurate 	 Accessibility Strategies to Consider Allow students to use a number line Provide a multiplication chart (black out the facts that the student knows) Ask students to find patterns in the facts Use software programs, such as FASTT Math, to build fluency Allow the use of calculators
Carry out algorithms and procedures	 Does not remember sequence of steps in an algorithm or procedure Leaves out key parts 	 Provide a model of worked-out calculations, highlighting the steps Provide practice problems and examples Teach mnemonic devices Have students create and use checklists Allow the use of calculators
Perform mental calculations	• Cannot keep the steps of a calculation in his or her working memory	 Allow students to use pencil and paper Have students talk about which operations they would use instead of calculating Allow the use of calculators
Solve multi-step problems	 Does not have needed information in his or her working memory to solve a problem Loses track of where they are in solving a multi-step problem 	 Provide resource sheets Provide templates or organizers to help students organize their work and keep track of the steps Teach students strategies for breaking problems into smaller parts

Automaticity: rapid recall of information with little or no effort. This is linked to fluency.

Use previously- taught skills and concepts	• Does not remember skills and concepts that were taught earlier in the year or in previous years	 Use a notebook organization system to help students find information in their prior work Review the needed skills at the beginning of the lesson or in the resource room Provide resource sheets or have students create their own reference books.
Use math vocabulary	 Has difficulty remembering math vocabulary 	 Preview the needed vocabulary Have students look up vocabulary words and write the definitions in a vocabulary section Use a word wall Provide a word bank of vocabulary terms for students to choose from
Show what they know on assessments such as quizzes and tests	 Performs poorly. Tends to do better on classwork than on assessments. Works slowly due to difficulty recalling information 	 Provide opportunities to review material before the assessment Provide alternative ways for the student to show what he/she knows Allow the use of reference Sheets, student resource books, and calculators Provide ample time for students to recall information during tests
Follow multi-step directions	 Forgets directions and is unsure what to do 	 Provide visual as well as auditory directions Ask students to paraphrase directions Help students highlight and underline key information in the directions of problems
Read and understand word problems	 Forgets what he/she is reading 	 Teach highlighting and underlining strategies. Suggest that students reread problem Ask students to restate problem in own words
Write math explanations	 Difficulty remembering math vocabulary terms to use Difficulty holding multiple pieces of information in mind in order to write a multi-step explanation 	 Provide a word bank of vocabulary terms for students to choose from or have the generate a list of words to use before writing sentences Provide a writing template
Take notes	 Notes are inaccurate and/or are missing information 	 Provide copies of notes or partially-completed versions for students to fill in
Answer questions in class discussions	 Responds slowly Anxious about making errors due to memory retrieval problems 	 Use ample wait time when calling on students Use a non-verbal signal to indicate that you will be calling on a student shortly
Recall information about math concepts and procedures	 Difficulty finding information in long-term memory Retrieves incorrect information 	 Change recall tasks into recognition tasks Present information in more than one format Point out which information is important by using visual and verbal cues Make explicit connections to prior knowledge to help students "file" new information Have students sort and categorize information to help them with storing and retrieving information in memory

Attention

In middle school, the increasingly complex math content and tasks lead to demands for longer attention spans from students. They need to complete multi-step investigations and long-term projects, pay attention to details, and complete tests and assessments, often within limited time. Students have to listen to directions and explanations, filter out extraneous information, participate in class discussions, and work effectively by themselves.

Attention		
Task Demands	Examples of Student Difficulties	Accessibility Strategies to Consider
Attend to Directions and Presentations by Teacher and Other Students	 Inattentive Tunes in and out Misses important directions and information. Make errors and has gaps in knowledge due to inattention Difficulty sitting still 	 Highlight relevant information by using color and by using verbal cues and vocal alterations in tone, speed, and use of pauses Use non-verbal signals to help focus attention. Focus attention on the "big picture" and the essential elements as opposed to giving additional details Use large visuals and written directions so that students do not need to rely solely on auditory processing.
Work on Math Tasks	 Difficulty sustaining attention for long repetitive tasks. Low productivity and difficulty completing class work and homework Easily frustrated and has low persistence. Responds to failure with decreased effort. 	 Start with easier tasks and move to more difficult. However, if tasks are too easy – students become bored and inattentive. If tasks are too difficult, the same issues may arise. Reduce the number of problems and the amount of repetition in large assignments. Break large tasks into smaller chunks. Sequence problems to provide opportunities for success and to build on success.
Solve Math Word Problems	 Makes frequent errors, especially when problems include irrelevant information Misses important details Jumps in to solve problem without considering different actions. 	 Have students talk about the problem first (with a partner) before starting to write answers. Often students make fewer errors when they talk first than when they write first. Reduce the amount of verbal information Teach strategies for students to identify and cross out irrelevant information

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Task Demands	Examples of Student Difficulties	Accessibility Strategies to Consider
Solve Word Problems, con't.	 Has difficulties when problems involve working memory, such as problems with mixed operations Gets lost in the middle of solving a problem 	 Use real-world problem contexts (helpful as long as problem length is restricted) Use problems that involve divergent thinking (out-of-box thinking, creative thinking) Give opportunities to communicate solutions and be persuasive Decrease the length of a task or the number of tasks
	 Difficulty categorizing problems by underlying structure. This can make it difficult to transfer prior knowledge to new problems. 	 Provide instruction on categorizing problems to help students see the deeper underlying structure. Help students to see beyond the surface elements of a problem.
Perform Basic Skills Calculations	 Lower computational fluency than peers Difficulty sustaining attention during repetitive tasks could contribute to difficulties automatizing basic computational skills Tends to make more errors on rote or overly familiar tasks. 	 Use a variety of instructional activities to facilitate fluency of basic calculations. Use computer software that provides immediate feedback. For some students, adding an element of competition – try to best their own scores may be motivating for doing a repetitive task. Use peer tutoring to increase engagement through social interaction Increase external stimulation (non-vocal music in background, color or movement added to problem Provide a multiplication chart. As students master certain facts, they should blacken them out showing progress.
Management of Time & Materials	 Difficulty with time management: working too quickly or too slowly Difficulty finding materials and tendency to lose things at school Difficulty finding completed homework (and thus not turning it in) 	 Provide time management cues Use timers Teach organizational strategies Post homework in same place daily so students can access: use online homework sites if possible Use notebook/binder organizational systems and do binder checks Provide a homework checklist

Attention, continued

Social-Emotional

Standards-based mathematics places a strong emphasis on the communication of mathematical ideas through classroom discourse. Students work together in pairs or small groups to carry out mathematical investigations and then share their findings in a whole class discussion. They may give their peers constructive feedback to help them improve a problem solution or project report. Students need confidence to try new mathematical investigations, to persist through frustration, and to share their ideas in public. All these types of tasks involve psycho-social skills.

Some students may misread social cues and thus cause tensions when they are working with peers. They may make inappropriate comments and disrupt class discussions. In these cases, teachers need to select small groups with care and set up structures for collaborative work and participation in discussions. Other students may give up easily on tasks because they are easily frustrated or lack confidence in their math abilities. These students may benefit from getting frequent feedback and from building on prior successes.

Social-Emotional		
Types of Tasks	Example Student Difficulties	Accessibility Strategies to Consider
Work in pairs or groups	• Finds that peer relationships cause tensions because of weak social skills	 Set clear expectations for student collaboration and individual accountability in small groups Choose groups with a specific purpose in mind: sometimes to mix skill levels, sometimes to promote particular social interactions
Move through a frustration point	 Strays from the concept being learned because of frustration, and focuses instead on lack of understanding Gets frustrated easily Lacks confidence Fears failure 	 Connect a new concept to one with which students have experienced success Check to make sure students have the necessary prerequisites Provide additional support time Provide frequent feedback
Play math games that involve winning and losing Give and receive constructive feedback	 Focuses too much on competitive aspect Is overly sensitive to losing Does not know how to give constructive feedback to peers Is overly sensitive to negative feedback 	 Minimize the competitive aspects of the game by presenting it as an experiment Make careful choices in assigning partners and teams Model giving and receiving constructive feedback Use a structured feedback process and provide handouts for the students to fill out

Fine-Motor

Fine-motor skills require precise, coordinated movements of the fingers and hands. These skills are needed to carry out a variety of mathematical tasks including performing calculations, writing explanations, making tables and graphs, using manipulatives, drawing representations, cutting out shapes, and building scale models. Students with fine-motor problems have difficulty with tasks such as aligning numbers, plotting points on graphs, and drawing straight lines. They tend to work slowly, and their final products may be illegible or lack the necessary precision. These students may benefit from having additional time for tasks and from using prepared templates that minimize the amount of drawing or cutting needed. Some students with physical disabilities may need alternative means for approaching fine-motor tasks, such as using software graphing programs, using software versions of manipulatives, recording their explanations, or working with a partner who is responsible for writing and drawing.

Fine-Motor		
Types of Tasks	Example Student Difficulties	Accessibility Strategies to Consider
Write explanations, solutions, or reports	 Does not write well by hand, particularly producing lengthy written responses Writes slowly Writes illegibly 	 Allow students to report answers orally Have students work in pairs or cooperative groups with a designated recorder Extend the amount of time for task Allow students to use a computer or portable keyboard to type their answers Allow students to record answers on a tape recorder
Create a graph, table, chart, and so on	 Does not draw these forms adequately Draws slowly Cannot draw straight lines and position them correctly Cannot write numbers small enough for small grids on graph paper 	 Provide templates for forms, either blank or partially filled-in Use larger grids Provide finger grip or nonskid rulers to give better control Allow students to use a spreadsheet program that creates the template
Align numbers	 Misaligns numbers when performing written calculations 	 Provide paper with vertical lines or place lined paper sideways Allow students to use graph paper
Use manipulatives	• Has difficulty moving, making patterns with, building, or aligning concrete manipulatives	 Provide a range of sizes and shapes of manipulatives Have students work in pairs or cooperative groups with a person designated to move manipulatives Use nonskid matting on desks to prevent sliding Use computer software that emulates manipulatives, if available
Cut	 Has difficulty using scissors Unable to cut accurately Cuts slowly 	 Provide precut pieces Have students work in pairs with one student cutting
Copy information or problems from the text or board	 Writes slowly Misaligns numbers Copies inaccurately 	 Provide handouts to minimize copying Reduce the number of problems to be copied