The importance of units and iteration in measurement

In order to become proficient in measuring space (in terms of length, area or volume), children need to understand two fundamental concepts: the idea of a unit and the process of iterating a unit to complete a measurement.

Understanding Units

The important aspects of learning about *units* include:

Appropriateness: A unit must be related to the aspect of the object to be measured. For example, you need a unit of area to measure area; you can't measure area with a linear tool. This distinction may not be easy for students, since they are often taught that calculating the area of a rectangle is as easy as measuring its length and width with a linear tool and multiplying the results. This procedure only makes sense, however, if the student understands that this calculation is equivalent to dividing the rectangle into identical *square* units and counting them. The idea of a unit of area – e.g. a square inch – is unfamiliar to many students, so they don't have a ready strategy for measuring area by counting square units. Several researchers have described young students' difficulty measuring area (Nitbach and Lehrer, 1996), demonstrating how they sometimes treat length as a space-filling attribute, measuring the area of an object in linear inches, not in square inches.

Consistency: Once you choose a unit, you must continue to use the same unit throughout your measuring task. The idea of maintaining a consistent unit in linear measurement is easier for children to understand than the parallel idea in measuring area. Since students often don't have a unit of area that has the same concrete status to them as an inch, it is difficult for them to be consistent about using the unit. If students are measuring area with pattern blocks, for example, they often use a variety of shapes to cover an area and then calculate the area by counting the total number of blocks without regard to how many units each shape represents.

Flexibility: Units do not have to be the standard ones, like inch, centimeter or square inch, that you would find in textbooks. Many commonly-used units are based on body parts (how many paces from here to there) or manipulatives (how many square tiles cover this area)? While units don't have to be standard, they do have to be agreed upon by a group of measurers and used consistently by each individual. For example, an arm span is not a useful unit unless it is the arm span of a specific person, and everyone who is measuring agrees to use that particular arm span.

Equivalence: You don't need to have a person present to use their arm span as a unit; you can use an equivalent length of some other material to "stand in" for their arm. Equivalence promotes the idea that the important aspect of a measurement unit is not the item that is being used as a unit (e.g., and arm in the example above), but rather the amount and quality of the space that the unit is taking up.

Tools as units: There are measurement tools that are equivalent to more than one unit. For example, a ruler measures 12 inches where the inch is a unit. This equivalence can be confusing for students if they don't understand the mapping of units to tools. They might consider a ruler as "1," which would be appropriate if the ruler had been chosen as the unit of measurement, but not if inches had been chosen.

Learning to Iterate

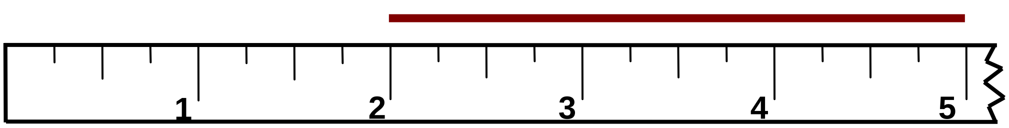
Once children choose an appropriate unit for measuring an item, they must consider how to *iterate* using that unit. Iterating a unit entails subdividing a space (linear, area or volume) according to the unit and counting subdivisions.

In this example, a student has chosen to measure the length of a picture frame with paperclips. By iterating the paperclips correctly, the student finds out that the length of the frame is 6 paperclips. Notice that the paperclips are lined up end to end with no gaps between them.

In this example a student is not iterating correctly. There are gaps between the paperclips, so the entire length of the picture frame is not being subdivided correctly. This causes the student to measure the length of the picture frame as 5 paperclips.

When they are younger, students may have trouble with any of the following aspects of unit iteration:

<u>Reading measurements from tools</u>: Even when a student has chosen an appropriate unit and an appropriate tool with which to measure an object, there are significant mathematical concepts that he or she must apply in order to correctly read the tool. Young children tend to think of a measurement as a position on a ruler, rather than as a measure of the space between the beginning of the ruler and that position. This can lead to trouble when they are measuring a length that begins at a point other than "0" on a ruler. It is notoriously difficult for students to correctly measure a line that extends from 2 to 5 on a ruler. Many say 5; others count the inch markings and say 4.



<u>Covering</u>: Learning to "cover" objects is also an important part of learning to iterate correctly. When measuring a length or an area, there cannot be any space that is not "covered" by a measurement unit. In particular, this means that if you are iterating with rulers, you need to put the end of the first ruler and the beginning of the second ruler right next to one another, so there is no gap in the progression of inches. Completely covering an area is generally more difficult than covering a length, primarily because it isn't as clear how to choose and arrange units, especially if students do not have a sense of "square" units. Children often leave small holes or use inconsistent units when they are covering an area; they would be much less likely to do so when measuring a length.

<u>Remainders</u>: Leftover space must be described with respect to the unit. The idea of a measurement taking up a fraction of a unit in length is relatively straightforward (although see above comments on the complexity of reading measurements from tools); students are not usually bothered by the idea of 3 ½ inches. In contrast, because students

are not as comfortable with the idea of a square unit of area, it is often more difficult for them to imagine a measurement that includes a fraction of a unit. If they are using square tiles, for example, and there are some “holes” left uncovered, they may use different shapes to cover them, rather than considering how they might be combined to create a measurement in terms of square tiles.

Reciprocity: The larger the unit, the fewer units it will take to measure a given aspect. For example, a student who uses yardsticks to measure the length of a room will require fewer yardstick-units than a student who measures the same room with 12-inch rulers. When children are quite young, they may equate a large unit with a large number of units, and will misapply this idea so that they think that the object being measured with the larger unit will automatically be the larger object. Later, children often come to understand the reciprocal nature of the unit-length vs. number-of-units relationship when they compare the results they get when they measure a line with long vs. short feet (perhaps belonging to different people in the class). These measurement-based ratios may be an accessible example of proportional reasoning for students in elementary school.

References

Nitabach, E. and Lehrer, R. (1996). Developing spatial sense through area measurement. *Teaching Children Mathematics*, 2, 473-476.